"MATHEMATICAL PROGRAMMING ON DIFFERENT TYPES OF OBJECTIVE FUNCTIONS"

विभिन्न प्रकार के उद्देश्य फलनों पर गणितीय प्रोग्रामन

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(**English)** **Sangeeta**

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2. **Title of Research Work** **Mathematical Programming on different types of Objective Functions**

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4. INTRODUCTION, IMPORTANCE OF PROPOSED INVESTIGATION, OBJECTIVE:-

4(A) Introduction

Programming problem is a planning problem which formulate as optimize a profit/Cost function with certain decision variables. These decision variables were restricted to satisfying a system of constraints.

4(A-1) Linear Programming Problem:

Linear programming is a main branch of operations research. In this type of programming problem, objective function is linear. Linear Programming= Linear+Programming, the term Linear implies that all the relationship involved in the problem are Linear and the term 'Programming' means the process of determining plan of action using Mathematical Technique.

Linear Programming problems is formulated as maximizing/minimizing a linear form of profit/cost function whose variable were restricted to values satisfying a system of linear and non negative constraints.

4(A-2) Linear Fractional Programming Problem:

In this type of problem objective function is in the form of ratio $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are linear functions. Problems of LFP arise when there appears a necessity to optimize the efficiency of some activity: profit gained by company per unit of expenditure of labor, cost of production per unit of produce goods, etc. Now a days, because of a deficit natural resources, the use of specific criteria becomes more and more typical. So an application of LFP to solving real world problems connected with optimizing efficiency could be very useful.

4(A-3) Multi-Objective Function:

In multi objective function, objects are more than one which may be linear or fractional or in their combination. To solve multi objective functions by various techniques, is very important for decision making and used in variety of problems having multiple objectives such as profit/cost, actual cost/standard cost, debit/equity, inventory sales etc. subject to the system constraints.

As an example of multi objective problem in daily business, suppose that we are designing a photocopy machine. A good design of product could mean a business success whereas a poor design may be fatal to corporate survival. Each design involves the cost of production, the speed of coping, the contrast of copy, the cost of using machine, etc. Then there is problem to determine a "best" combination of values of Objectives, like

$$\text{Min}\{cost\ of\ production\}$$

$$Max\{the\ speed\ of\ copying\}$$

$$Max\{the\ contrast\ of\ the\ copy\}$$

$$Min\{the\ cost\ of\ using\ the\ machine\}$$

for business success.

4(A-4) Inverse Objective Value Optimization Problem:

In this type of problem we do not have the desired solution in advance, but we do have the desired value of the objective function we wish to adjust the parameter values which make the optimal objective value equal to its desired value.

4(B) Importance of Proposed Investigation

The proposed research work has a great importance in itself. New research in the theory of subject will give an easy and efficient approach towards the solution of more complex problems arising in Management, Defense, Government, etc. so that the significance of the study may be useful for the economic growth and national development.

Interest in fractional programming is generated by the fact that various optimization problems from engineering and economics consider the minimization of a ratio between physical and/or economical functions, for example cost/time, cost/volume, cost/profit, or other quantities which measures the efficiency of a system. For example, the productivity of industrial systems, defined as the ratio between the realized services in a system within a given period of time and the utilized resources, is used as one of the best indicators of the quality of their operation. Such problems, where the objective function appears as a ratio of functions, constitute fractional programming problem like modeling various decision processes in management science, operational research, economics and other fields.

Inverse Optimization plays an important role in geophysical science, medical imaging, traffic equilibrium, location problems, production planning problems and such these types of problems. For example in transportation network, users generally select the route (flow) that minimize their cost (or time), this type of flow in a network is called user equilibrium flow. If a transportation planner wants to enforce the users to use a particular flow (route) then this flow is called the system optimal flow. The user equilibrium flow may or may not be equal to the system optimal flow, in the case when these are not equal, a minimum toll can be

imposed on some road segment of the route, so that the user equilibrium flow become identical with the system optimal flow.

Multi-Objective Linear/ Fractional/Linear plus Fractional Programming problem is a very important technique for decision making and is used in variety of problems having multiple objectives such as profit/cost, actual cost/standard cost, debt/equity , inventory sales etc, subject to the system constraints. It can also be used to solve various real life problems related to fields like planning problems in agriculture, production, inventory etc.

4(C) Objective

The main object of the proposed research work is to study the various methods for the solution of different type of objective functions like Linear, Fractional, Linear plus Linear Fractional, Multi-Objective functions, for this we will apply various type of techniques like Inverse Optimization etc.

Our objective is to find some alternative methods to solve Linear, Fractional, Linear plus Fractional or Multi-Objective problems which is less time consuming and easy to solve (or easy Solution) and useful for society.

We can also verify, if a method is applicable for linear programming and for fractional programming then either it is also applicable for Linear Plus Fractional programming problems or not and how to improve it.

LITERATURE REVIEW:

The first problem in transportation or Linear Programming was developed by Kantorovich, a Russian Mathematician and American economist Hitchcock. They dealt with a well known transportation problem which form a branch of Linear Programming. Even through a French Mathematician Jean Baptist Joseph Fourier seemed to be aware of the subject potential. Kantorowhich published an extensive monograph on mathematical methods in the organization and planning of production in 1940.

The practical applications of Programming Problems came in 1947, when George Danzig (American mathematician) devised the Simplex algorithm for solving General Linear Programming Problems while he was a member of group which, under the direction of Marshall Wood, was working on certain types of Programming Problems for the United State Air Force. It provided the real spark for the phenomenal growth of interest in the theory and application of Programming Problems. Dantzig published a paper on Linear Programming in 1948. Further Beale, Doig, Dantzig, Land and other mathematicians published their research papers on Linear Programming.

Linear Fractional Programming Problems were introduced by Charnes and Cooper in 1962. These types of problems consist of Linear type fractional Objective Functions and linear constraints. Other authors who contributed in this field were Dorn (1962), Zionts (1968) , Bitran and Novaes (1973) Magnanti (1976), Schnabel (1981) and Schnabel (1983).

Charnes, Cooper and Henderson (1963) published a paper "An Introduction to Linear Programming". They used Mathematical Programming Problems to solve optimization problem. Dowing (1992) advocated that the lagrangian method should be used for any optimization subject to a single inequality constraint and Dwivedi (2008) posit that Linear Programming is of great use in making business decision because it helps in measuring complex economics relation and there by , provides an optimum solution to the problem of resource allocation.

In 1965, Ijiri gave a note on Multi Objective Programming in his paper "Management Goals and Accounting for Control". In this paper author introduced the Multi objective programming problems arising in Management and Accounts. Johnson (1968) studies "Multi Objective Decision Models". He described various models of Multi Objective Programming Problems. Roy (1972) published paper on "Problems and Methods with Multi Objective Functions". In this paper Roy introduced some initial methods to solve Multi Objective Programming Problems. Cohon and marks (1973) gave lecture note in "A Review and Evaluation of Multiple Objective Programming Techniques". In this paper author defined some special techniques using utility function to solve problems. In 1974 Zeleny gave a

lecture note on “Linear Multi Objective Programming” Zeleny gave brief overview of Linear Multi Objective Programming Problems and its applications.

Afterward Ignizio in 1976, published a paper on “Goal programming and extensions” and in this paper he gave relation between multi objective programming and Goal Programming Problems. Cohon (1978) with “multi objective programming and planning” and Cohon gave application of multi objective programming in production planning problems arising in Industries.

Inverse Optimization is a relatively new area of research in the past few years. The operations research community have shown a significant interest in the field of Inverse Optimization and a variety of inverse optimization problem have been studies by researchers. Geophysical scientists were the first ones to study Inverse Optimization problems. A very useful contribution on Inverse Optimization was Inverse linear programming problems, given by Zhang and Lui. Burton and Toint (1992) published a paper “On an instance of the inverse shortest path problem”.

In 90s decade, research work on multi objective programming had extended towards Fractional Programming problems and various methods for solutions had given. Jain (2006) published a paper on “ Solution of multi objective Fractional Programming”. In this paper author took multi objective programming problems with Fractional form of objective functions and gave solutions procedure.

Tantawy (2007) published a paper “ A New Method for Solving Linear Fractional Programming Problems”. The proposed method is based mainly upon solving this problem mainly algebraically using the concept of duality. In 2008, Tantawy again published a paper “An Iterative method for Solving Linear Fractional Programming Problem with sensitive analysis”.

Chergui and Maulai (2008) Developed a branch and cut algorithm based on continuous Fractional Optimization, for generating the whole integer efficient solution of the Multi Objective Linear Fractional Programming Problem in “ An exact method for a discrete Multi Objective Linear Fractional Optimization”.

Effati and Pakdaman (2012) introduce an Interval Valued Linear Fractional Programming Problem (IVLFP) in his paper "Solving the Interval Valued Linear Fractional Programming Problem".

Borza, Rambely and Saraj published a paper "Solving Linear Fractional Programming Problems with Interval coefficient in the objective functions, a new approach". In this paper,the Linear Fractional Programming Problem with interval Some results in interval multiobjective linear programmingfor recognizing different solutions. S. Rivaz and M. A. Yaghoobi(2015) published a paper "Some results in interval multiobjective linear programming for recognizing different solutions". This paper introduces a new solution concept namely necessarily weak efficient.

Mahesh N. Dumaldar published a paper "On efficient solutions of 01 Multiobjective linear programming problems" which states that at least one optimal solution among the optimal solutions of an individual 01 linear program is an efficient solution.

Chakraborti (2015) proposed "Evolutionary technique based goal programming approach to chance constrained interval valued believe programming problems". Here auther discussed, Many decision making problems often need to be modeled as a class of bilevel programming problems with inexact coefficients and chance constraints. To deal with these problems, a genetic algorithm based goal programming procedure for solving interval valued bilevel programming problems in a large hierarchical decision making and planning organization is proposed. In the model formulation of the problem the chance constraints are converted to their deterministic equivalent using the notion of mean and variance. Further, the individual best and least solutions of the objectives of the decision makers located at different hierarchical levels are determined by using GA method. The target intervals for achievement of each of the objectives as well as the target interval of the decision vector controlled by the upperlevel DM are defined.

Recently Abhijit Baidya, Uttam Kumar Bera and Manoranjan Maiti (2016) published a paper on "The grey linear programming approach and its application to multiobjective multistage solid transportation problem". The aim of this problem is to obtain the optimum distribution of goods from different sources to different destinations with different mode of conveyances which minimizes the total

transportation cost and time. But it may contain one or more stage to transport the commodities with different mode of transport.

RESEARCH GAP

The main origin of Operation Research was during the Second World War. At that time, the military management in England called upon a team of scientists to study the strategic tactical problems related to air and land defense of country. Since they were having very limited military resources, it was necessary to decide upon the most effective utilization of them.

The first mathematical technique in this field was developed in 1947 by American mathematician, George B. Dantzig who gave Simplex Method of linear programming. Since then, new techniques and applications have been developed through the efforts cooperation of interested individuals in academic institutions and industry both. The rudiments of the linear programming had emerged at mid of 20^{th} century in the investigation of Kantorovich in 1940, Dantzig 1947, Charnes and Coopers 1962.

In 1956, J.R. Issbel and W. H. Marlow discussed an example of Fractional Programming. After that, Operation Researchers took interest in Fractional Programming. The development of Fractional Programming was come into light after 1967, when Zionts, Bitran and other mathematicians published their research papers. During the period of 1968-1983 many research papers have been published on Fractional Programming. After 1967, we found less work on Linear Programming.

We found research work on Linear plus Linear Fractional Programming Problems after 1993. Multi-Objective Mathematical Programming has been one of the fastest growing area of O.R. during the last twenty years.

We can see many research papers on various mathematical programming with Fuzzy approach, but I found less work with linear factors in Multi Objective, Fractional, Linear Plus Linear Fractional Programming. Most recent works in this field are of Tantawy, Mehdi, Jain, Mangal, Rambely and many other mathematicians.

HYPOTHESIS

A hypothesis is a speculation or theory based on insufficient evidence that lends itself to further testing and experimentation. With further testing, a hypothesis can usually be proven true or false.

Null Hypothesis :

A null hypothesis is a hypothesis that says there is no statistical significance between the two variables in the hypothesis. It is the hypothesis that the researcher is trying to disprove.

Alternative Hypothesis:

An alternative hypothesis simply is the inverse, or opposite, of the null hypothesis.

Hypothesis for our research :

As various operation techniques are applied on Linear, Fractional Programming Problems and we get an optimal solution, similarly different operation techniques can also be applied on Linear plus Linear Fractional, Multi Objective Programming Problems and we can get optimum solution.

To study techniques and methods of Mathematical Programming such that a Multi Objective Programming Problem can be reduced into Fractional Programming Problem and by substituting its denominator as 1 we may convert it into Linear Programming Problem, so that it can be applicable for all reducing forms of Mathematical Programming.

Here we assume, all the functions involving in all Programming Problem are Linear.

as in Fractional Programming Problem:

$$F(x) = \frac{P(x)}{Q(x)}$$

and in Multiobjective Programming Problem:

$$Z(x) = \max\ (or\ \min)\{P_1(x), P_2(x), \dots, P_n(x)\}$$

and in Linear plus Linear Fractional Programming Problem:

$$Z(x) = R(x) + \frac{P(x)}{Q(x)}$$

where $P(x), Q(x) \neq 0, R(x), P_1(x), P_2(x), \dots, P_n(x)$ are linear functions.

RESEARCH METHODOLOGY

A Linear Fractional Programming Problem can be formulated mathematically as follows, and will be denoted by (LFP)

$$Maximum\ Z = \frac{\gamma^T + \alpha}{\delta^T + \beta}$$

subject to the constraints:$Ax = b, x \geq 0,$

where $x, \gamma, \delta\ are\ n \times 1$ column vectors, $\delta^T + \beta \neq 0$, A is $m \times n$ matrix, b is $m \times 1$ column vector, $\alpha\ and\ \beta$ are some scalars. Further we assumed that the constraints set $S = x; Ax = b, x \geq 0\}$ is non-empty and bounded.

The Multi-objective Linear plus Linear Fractional Programming Problem (MOLPLFPP) can be formulated as:

Maximize $Z_i(x) = (c_i^T + d_i) + \frac{\gamma_i^T + \alpha_i}{\delta_i^T + \beta_i}$

subject to $x \in S = \{x \in R; Ax \leq = \geq b, x \geq 0$

$$i = 1,2,3, \dots, k.$$

where $c_i^T, \gamma_i^T, \delta_i^T$, and A$\in R^{N \times M}, b \in R^M$, S is assumed to be non-empty, convex and compact in R^N and further we also assume that x $\in$S and $(\delta_i^T + \beta_i) > 0\ , i = 1,2,3, \dots, k.$

Here we can see that the objective function, $Z_i(x)$ in this problem is a combination of two terms. The first term is linear and the second term is fractional with linear numerator and denominator.

The study of multi objective problems consists a development of new methods and implementation in methods so that current complex problems arising in Industries Government, etc. can be solved. In general way we can address the multi Objective Programming Problems in Mathematical form as:

$$Max\{f_1(x) = z_1\}$$

$$Max\{f_2(x) = z_2\}$$

:

:

:

$$Max\{f_k(x) = z_k\}$$

s.t. $x \in S$

where $S = \{x \in R^N; Ax = b, x \geq 0, b \in R^M\}$

which is linear. A multiple Objective Problem is just like a single Objective Problem except that it has a stack of objective functions instead of only one.

Formulation of Inverse optimization problem:

Let us consider the following optimization problem

$$\min \ f(c, x)$$

$$s.t. \ \ x \in S$$

Where $S \in R^n$ is the set of feasible solution, $c \in R^n$ a given cost vector. If $x' \in S$ is the given feasible solution, $c' \in R^n$ is the perturbed value of c and $\| \, . \|$ be some norm then its Inverse is

$$\min \|c' - c\|$$

$$s.t.\, f(c', x') = \min\{ f(c', x) : x \in S\}$$

$$l \leq c' \leq u \qquad and \quad c' \in R^n$$

and other various methods can be used in our research work.

In our research methodology, we may

- Apply various type of methods which is easier and less time consumed in comparison to existing methods.
- Add some new parameters in mathematical programming to develop new method.
- Use applications related to hospitals, industries and society.

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